 DOI: 10.1086/673914

Appendix D from H. Uecker et al., "Evolutionary Rescue in Structured Populations"

(Am. Nat., vol. 183, no. 1, p. E17)

Island Model without Standing Genetic Variation

We here restrict ourselves to $\alpha = 0$ (i.e., mutants are lethal in the old environment) and $\beta = 0$ (no additional density dependence beyond the hard carrying capacity). In the main text equation (14), we derived the following effective growth parameter of a mutant:

$$s_{\text{eff}}(t) = \begin{cases}(1+s)(1-m) + (1+s)m\dfrac{d}{D} - 1 & \text{for } t \in [(d-1)\vartheta, d\vartheta[,\ d \in \{1, \ldots, D-1\},\\ s & \text{for } t \geq (D-1)\vartheta.\end{cases} \tag{D1}$$

Using this, we can calculate the establishment probability with equation (A7), setting again $\Phi = \vartheta$. For the total rescue probability, we obtain (cf. eq. [5]):

$$\begin{aligned} P_{\text{rescue}} \approx 1 - \exp&\left[-u\sum_{t=0}^{(D-1)\vartheta-1}(1+s)\left(1-m+\frac{d_t}{D}m\right)N_{\text{w}}^{(\text{new})}(t)p_{\text{est}}(t+1)\right]\\ &\times\ \exp\left[-u\sum_{t=0}^{(D-1)\vartheta-1}(1+s)mK(D-d_t)\frac{d_t}{D}p_{\text{est}}(t+1)\right]\times\exp\left[-u(1+s)\frac{N_{\text{w}}^{(\text{total})}((D-1)\vartheta)}{r}\frac{2\hat{s}}{1+\hat{s}}\right],\end{aligned} \tag{D2}$$

where $N_{\text{w}}^{(\text{total})}((D-1)\vartheta)$ is the wildtype population size immediately after the last deme has deteriorated. The first term takes mutants into account that originate in the new part of the habitat. The second term considers mutant offspring of individuals from old demes that migrate to the new part where they can survive. The last term is the same as in the Levene model. As $\alpha = 0$, there are no mutants in the population before time $t = 0$.

In the main text, we gave an approximation for the probability of evolutionary rescue for $D = 2$ (see eq. [17]), which generalizes to more than two islands in a straightforward way. The stationary value $\hat{N}_{w,d}^{(\text{new})}$ of wildtype individuals in the perturbed part of the habitat in a period with d deteriorated demes is obtained as the solution of

$$0 = (1-r)\left(1-\frac{D-d}{D}m\right)\hat{N}_{w,d}^{(\text{new})} + m\frac{d}{D}(1-r)(D-d)K - \hat{N}_{w,d}^{(\text{new})}. \tag{D3}$$

This yields

$$\hat{N}_{\text{w},d}^{(\text{new})} = \frac{d(D-d)Km(1-r)}{(D-d)m(1-r)+Dr}, \tag{D4}$$

and with

$$2s_{\text{eff}}(t) \approx 2\left(s-m+m\frac{d_t}{D}\right), \tag{D5}$$

we obtain

$$P_{\text{rescue}} \approx 1 - \exp\left\{-u\vartheta\sum_{d=1}^{D-1}2\max\left[\left(s-m+m\frac{d}{D}\right),0\right]\frac{d(D-d)Km(1-r)}{(D-d)m(1-r)+Dr} - u\frac{DK}{r}2s\right\}. \tag{D6}$$

Figure D1 shows a comparison between the exact formula (D2) and the approximation. The approximation captures the behavior for small m very well. In particular, it reproduces the maximum in the probability of evolutionary rescue. As m increases, the approximation becomes worse.

If the number of demes D is large and $r \gg m$, we can approximate equation (D6) with

$$P_{\text{rescue}} \approx 1 - \exp\left\{-u\vartheta \int_0^D 2\max\left[\left(s - m + m\frac{d}{D}\right), 0\right] \frac{d(D-d)Km(1-r)}{Dr} \mathrm{d}d - u\frac{DK}{r}2s\right\},$$
$$= 1 - \exp\left\{-u\vartheta \frac{1}{3} D^2 Km \frac{1-r}{r} \max\left[\left(s - \frac{1}{2}m\right), 0\right] - u\frac{DK}{r}2s\right\}. \tag{D7}$$

From equations (17) and (D7), we find that the maximum is at $m \approx s$.

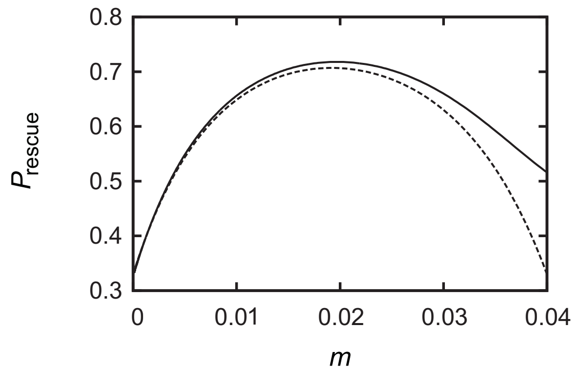

Figure D1: Evolutionary rescue in a two-island model. The plot compares the exact result (D2) (solid line) with approximation (17) (dashed line). The parameter values are the same as in figure 10.