Appendix A from L. P. A. van Gerven et al., “Competition for Light and Nutrients in Layered Communities of Aquatic Plants” (Am. Nat., vol. 186, no. 1, p. 000)

Species’ Minimal Resource Requirements and Resource Consumption Vector

The lowest required nutrient concentration $R_{\text{min}}$ for the plants studied follows from the requirement for persistence $p_i = m_i$ (see eqq. [1] and [2]) when light is nonlimiting ($f_i = 1$). Using equations (3) and (4) from the main text, this leads to

$$R_{\text{min},i} = \frac{m_i M_i}{p_{\text{max},i} - m_i}. \quad (A1)$$

The derivation of the minimal required light intensity $I_{\text{min}}$ is less straightforward, as $I_{\text{min}}$ is reached only when there is no self-shading and thus when the equilibrium biomass approaches zero. Furthermore, for submerged plants, $I_{\text{min}}$ also depends on the background light attenuation of the water. Here $I_{\text{min}}$ follows from $p_i = m_i$ when nutrients are nonlimiting ($f_a = 1$), and in case the background attenuation is zero, it follows from equations (3), (5), and (6) that

$$I_{\text{min},i} = \frac{m_i H_i}{p_{\text{max},i} - m_i}; \quad (A2)$$

as for this case, equations (5) and (6) read

$$\lim_{\text{biomass} \to 0} f_{ij} = \frac{I}{I + H_i}. \quad (A3)$$

The consumption vector can be derived by expressing the equilibrium biomass in terms of nutrients and light. For floating plants at monoculture equilibrium $F^*$, this means that $dF/dt = 0$ (eq. [1]) and $dR/dt = 0$ (eq. [9]), leading to a biomass expression in terms of nutrients:

$$F^* = \frac{r_i - z_i D R^*}{c_i m_F}, \quad (A4)$$

where the stars denote the equilibrium state. The equilibrium biomass $F^*$ can also be expressed in terms of light (following from eq. [7]):

$$F^* = \frac{\ln(I_{in}) - \ln(I_{out})}{k_F}. \quad (A5)$$

The consumption vector results from combining both expressions for $F^*$ (eqq. [A4] and [A5]) and substituting expressions for $r_i$ and $I_{in}$ based on the resource supply point, which represents the highest-possible resource levels in equilibrium (Tilman 1980) that are achieved in a system without plants ($I_{in} = I_0$ and $r_i = z_i D R$, following from eqq. [7] and [9]):

$$\ln(I_{in}) = \frac{k_F}{c_i m_F} z_i D (R - R^*) + \ln(I_{0i}). \quad (A6)$$

Thus, the consumption vector represents how the nutrient concentration $R$ and the light intensity at the lower end of the plant $I_{in}$ are reduced to their equilibrium levels due to plant consumption. Similarly, the consumption vector of the submerged plant can be derived, which leads to

$$\ln(I_{out}) = \frac{k_F}{c_i m_F} z_i D (R - R^*) + \ln(I_{0out}). \quad (A7)$$
The slope of the consumption vector in the \( \ln(I_0) - R \) and \( \ln(I_{out}) - R \) planes, respectively, which for both plants equals \( (k/c_m)z_B D \), indicates the factors that control plant resource consumption. As a result, the resource consumption is controlled by both environmental conditions (water column depth \( z_B \) and dilution rate \( D \)) and species traits (light attenuation coefficient \( k \), nutrient content per unit biomass \( c \), and loss rate \( m \)).